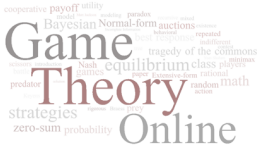
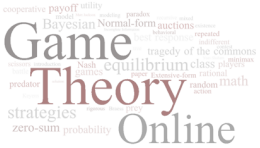


Analyzing Bayesian Games: Another Example

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Bayesian (Nash) Equilibrium



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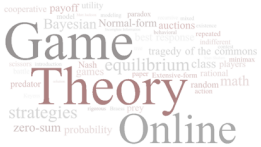

- A plan of action for each player as a function of types that maximize each type's expected utility:

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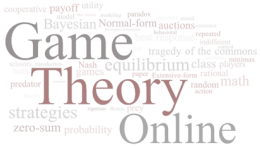

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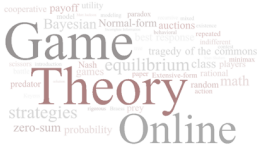

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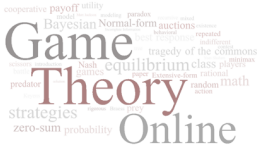
A Sheriff's Dilemma



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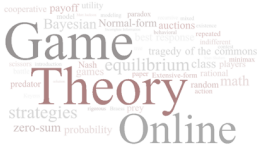

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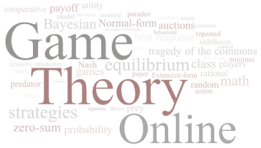

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- the sheriff would rather shoot if the suspect shoots, but not if the suspect does not.

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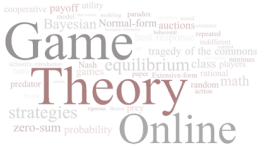

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- the criminal would rather shoot even if the sheriff does not, as the criminal would be caught if does not shoot.

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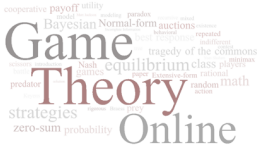

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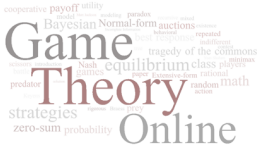
- the criminal would rather shoot even if the sheriff does not, as the criminal would be caught if does not shoot.
- the innocent suspect would rather not shoot even if the sheriff shoots.

A Sheriff's Dilemma

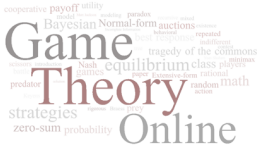


$Sheriff$

$Good$	$Shoot$	Not
$Shoot$	$-3,-1$	$-1,-2$
Not	$-2,-1$	$0,0$

Bad	$Shoot$	Not
$Shoot$	$0,0$	$2,-2$
Not	$-2,-1$	$-1,1$

A Sheriff's Dilemma



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Summary: Bayesian (Nash) Equilibrium

- Explicitly models behavior in an uncertain environment
- Players choose strategies to maximize their payoffs in response to others accounting for:
  - strategic uncertainty about how others will play and
  - payoff uncertainty about the value to their actions.